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Impulse response measurement technique for the analysis of the radiation efficiency of submerged circular plates

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ABSTRACT

The sound radiation of submerged structures is a topic of great interest both in the fields of underwater acoustics and marine engineering. In this context, the radiation efficiency is a very useful parameter to relate the underwater radiated noise and the vibration of a given structure. This paper investigates the average radiation efficiency of submerged circular plates using an impulse response measurement technique. For this purpose, thin plates made from different materials, steel and aluminium, and with either rigid or flexible boundary conditions (i. e. welded or using an O-ring) were manufactured. Each system was excited with an electrodynamic actuator, the vibration level and the underwater sound radiation being measured by using accelerometers and hydrophones, respectively. Experiments were performed in a water tank using a MLS technique along with a robotized positioning system under laboratory-controlled conditions. Additionally, results were compared with preliminary analysis data for air loading conditions, the main differences between both scenarios being discussed.

Keywords: Radiation efficiency, Impulsive response, water loaded circular plates, underwater acoustics, Maximum Length Sequence.

I-INCE Classification of Subject Number: 42

1. INTRODUCTION

The analysis of the vibrating structures in contact with water has special interest to control the Underwater Radiated Noise (URN) by any maritime platform and, in particular, to control the acoustic environment pollution generated by ship traffic.

Beams, plates and shells are the main elements used in the design of most maritime structures. The present research is concerned to thin circular plates.

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The radiation efficiency is a quantity that characterizes the efficiency of a given vibrating surface as a sound. The vibrations of circular plates in contact with water have been studied widely in the literature. The first fluid-structure interaction problem stemmed from the classical problems solved by Lord Rayleigh [1] and Lamb [2]. Lord Rayleigh [1] calculated the increase of inertia of a rigid disk harmonically moving in a circular aperture. By other hand, Lamb [2] studied the free vibrations of clamped, circular baffled plates by using simple assumed modes and an approximation to obtain the hydrodynamic pressure. This solution was extended to free-edge circular plates by McLachlan [3] and Peake et al. [4]. Powell et al. [5] conducted experiments to verify Lamb's theoretical results. Amabili and Kwak [6] solved the same problem by using the Hankel transform for a plate in contact with a liquid on one side and placed into the hole of an infinite rigid wall. Kwak [7] obtained the so called Added Virtual Mass Incremental (AVMI) employing the integral transformation technique in conjunction with the Fourier-Bessel series approach.

Kwak et al. [8] and Kwak [9] studied the free vibrations of circular plates resting on a free water surface. These studies also addressed circular plates completely submerged in an infinite fluid domain. Amabili et al. [10] performed experiments to confirm the results of [8-9]. Kwak et al. [11] extended this study to annular plates, successfully comparing theoretical and experimental results. In both, the plates were fully immersed in a cylindrical tank of 340 mm length x 500 mm diameter. The free-edge condition was obtained placing the plate on a compliant suspension made by low stiffness wires. The excitation was generated by means of a shaker and the measurements of the vibrations by means of a laser doppler vibrometer to guarantee not to load the structure and not to alter the fluid – structure interaction. The excitation signal was a random burst with 40% of the signal in the sample period. Espinosa et al. [12] performed experimental tests to verify theoretical results of a simple approximate method used to determine the influence of the fluid on the frequency applied to water-loaded circular plates vibrating in their axisymmetric modes. The measurements were carried out in a water tank 6.5 meters long, 4.5 meters wide and 4.5 meters deep. The circular plates were suspended from their nodal point by means of thin wires. The exciter was a small piezoelectric vibrator and the frequencies were determined by means of a hydrophone.

The water depth has influence on the natural frequencies. Kwak et al. [13] concluded theoretically and experimentally that the effect of the fluid depth can be neglected when the fluid depth is greater than the diameter of the circular plates but becomes significant as the water depth decreases. The measurements were conducted inside a water tank of 1,5 meters high and a diameter of 1,2 meters. The plate was supported by a string thus enabling the free edge boundary condition. To excite the plate, an impulsive hammer was used. The vibrations were measured by means of an accelerometer attached to the plate with negligible weight.

The effect of fluid loading on radiation efficiency has been studied by Davies [14-16], Berry [17] and Rumerman [18,19]. Rumerman studied the radiation efficiency of steel plates loaded with water showing that when the fluid loading is not light, the use of classical radiation efficiency overestimates the radiated power. Recently, Cheng et al. [20] introduced engineering formulas for radiation efficiencies of point-excited submerged rectangular plates.

The goal of this research is to present a method based on the Maximum Length Sequence (MLS) technique to measure the impulsive response of thin circular plates in contact with water and hence the surface velocity and acoustic responses. The experiments were conducted in a water tank with an automatic positioning system. The great advantage of this method is that provides accurate measurements with conventional instrumentation without the necessity of using shakers and laser trackers.

2. THEORETICAL BACKGROUND

2.1 Radiation Efficiency Definition

The radiation efficiency σ of a vibrating structure is defined as the ratio of the total radiated sound power by the structure to the spatially averaged mean square velocity of the radiating surface and can be expressed in a normalized form as

$$\sigma = \frac{W_{rad}}{\rho c S \langle \overline{v^2} \rangle} \tag{1}$$

where W_{rad} is the total radiated sound power, $\langle \overline{v^2} \rangle$ denotes the temporal and spatial average square of the surface velocity, S is the radiating surface area, and ρ and c are the density and sound velocity in the acoustic medium, respectively.

The acoustic power radiation can be defined as the rate of acoustic energy delivered by a source. Since the acoustic intensity is the acoustic power flow per unit area, the total acoustic power radiated by any source can be obtained by integrating the acoustic intensity over a surface of convenience.

There common approach used to determine the total acoustic power radiated by planar sources is the Far-Field integration or Rayleigh integration, which need the velocity distribution on the surface.

In the Far-Field approach, the radial component of the acoustic intensity is integrated over an imaginary far-field hemisphere enclosing the source. The Rayleigh surface integral is used to calculate the pressure, the acoustic intensity and hence the acoustic power radiation from a plate vibrating. Each elemental area on the plate surface is assumed to be a point source and their individual contributions are summed to yield the total acoustic power radiation. The time-dependent far field complex acoustic pressure, in spherical coordinates is given by

$$p(R,\phi,\theta,t) = -i\frac{\rho c k e^{-i(wt-kR)}}{2\pi R}\int_S u e^{-ikrsin(\theta)\cos(\phi-\alpha)} dS \tag{2}$$

where u is the transverse velocity distribution on the surface of the source, k is the acoustic wavenumber, w is the angular frequency of the vibration, r and α are the polar coordinates locating the centre of the point source and R, θ and ϕ are the spherical coordinates locating the far-field point. The assumption of far-field point requires R being very much greater than r.

Suppressing the time-averaged dependency, for planar progressive waves and in case of a harmonically vibrating source, the average of the intensity over all time is defined as

$$I(R,\phi,\theta) = \frac{|p(R,\phi,\theta)|^2}{2\rho c} \tag{3}$$

The total acoustic power radiation can be written in spherical coordinates as

$$W_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I(R,\phi,\theta)\sin(\theta)R^2 d\theta d\phi \tag{4}$$

where $I(R, \phi, \theta)\sin(\theta)$ is the radial component of the intensity, and $R^2 d\theta d\phi$ is the area of the infinitesimal surface element of the hemisphere.

The Far-Field integration approach requires the knowledge of the velocity of the vibrating source to obtain the total acoustic power and hence the radiation efficiency. Therefore, the experimental method shall measure simultaneously the surface vibration and the radiated acoustic pressure.

2.2 Maximum Length Sequence (MLS) Method

The impulsive response is the reaction of the vibrating plate to an external change and is used to derive all the acoustic parameters of the structure. A common method for measuring the impulsive response of an acoustic system is to apply a known input and measure the output. In a Linear Time-Invariant (LTI) system, the output signal $y(n)$ has the form of convolution of the input signal $x(n)$ and the impulsive response $h(n)$ as

$$y(n) = x(n) * h(n) \tag{5}$$

where $*$ denotes discrete linear convolution. When the input corresponds to the Dirac delta function $\delta(n)$, the results of convolving with any function is the function itself. Therefore, the impulsive response is the output of an LTI system excited by a Dirac delta function as

$$h(n) = \delta(n) * h(n) \tag{6}$$

There are two common methods to measure the impulsive response of a system; the direct method and the correlation method. The direct method is based on exciting the tested system with a short pulse to emulate the Dirac delta function. The Dirac delta is a mathematical artefact and cannot be generated in a real physics. The correlation method is based on using wide band random signals with spectrum characteristics similar to the Dirac delta function to excite the system.

The correlation method is based on the property that the cross-correlation of the input and the output of a Linear Time-Invariant (LTI) system is the auto-correlation convolving with the impulsive response. When the auto-correlation of the input corresponds to the Dirac delta function the result of the cross-correlation is the impulsive response.

$$R_{xy}(n) = R_{xx}(n) * h(n) = \delta(n) * h(n) = h(n) \tag{7}$$

where $R_{xy}(n)$ is the cross-correlation, $R_{xx}(n)$ is the auto-correlation and $h(n)$ is the impulsive response.

The Maximum Length Sequence (MLS) technique is a correlation method commonly used in acoustics based on binary periodic pseudo-random sequences of length $P = 2^N - 1$, where N is the order and P is the periodicity. The spectrum of the MLS pseudo-noise is flat everywhere except at direct current DC.

The impulse response is obtained by circular cross-correlation between the measured output and the MLS sequence. Due to the use of circular operations to deconvolve the impulse response, the MLS technique obtains the periodic impulse response $h'(n)$ as

$$h'(n) = \sum_{k=-\infty}^{\infty} \delta'(k) h(n-k) = \sum_{k=-\infty}^{\infty} h(n + kP) \tag{8}$$

where $\delta'(n)$ is the periodic Dirac delta. The periodic impulse response $h'(n)$ is the impulse response repeated at period P. The *Equation 7* reflects the time aliasing error of the MLS technique. This error is significant if the length P of one period is shorter than the length of the impulsive response to be measured. Therefore, the order N of the MLS sequence must be high enough to overcome the time-aliasing error.

The measured periodic output is obtained by means of the circular convolution of the periodic MLS sequence and the periodic impulse response as

$$y'(n) = x'(n) \otimes h'(n) = \sum_{k=0}^{P-1} x'(k)\, h'(n-k) \tag{9}$$

where $x'(n)$ is the periodic MLS sequence and $h'(n)$ the periodic impulse response. ⊗ denotes circular convolution. By deconvolving, the periodic impulse response is given by

$$h'(n) = \frac{1}{P+1}\left(\sum_{k=1}^{P} y(k)x(k-n)\right) \tag{10}$$

The matrix containing $x(n)$ is called the M-sequence matrix. When P becomes large, the correlation with the M-sequence matrix can be prohibitive. The computational time can be minimized if the M-sequence matrix is processed with the Fast Hadamard Transform (FHT). Borish et al. [21] and Xiang [22] detailed the FHT method to calculate the impulse response of a system excited with an MLS sequence. In this research, the FHT method was used to obtain the impulse response of the vibrating plates under measurement.

3. MATERIALS AND METHOD

This section describes the vibrating structures under measurement, the experimental setup and the followed procedure.

3.1 Structures under study

Three circular plates were manufactured, two made of steel and other of aluminium. The diameter of the structures was 50 cm and the thickness 2 mm. The circular plates had a circular ring of 15 cm height and 2 mm of thickness. The circular ring was used to allow the deployment of the plate during the experiment. Two ways to fix the circular base with the circular ring were used during the manufacturing process. One steel plate and the aluminium plate were manufactured using an O-ring between the circular base and the circular ring. For the other steel plate, welding was used to fix the circular base to the circular sides. Figure 1 shows a schematic diagram of the dimensions of the plates with and without O-Ring.

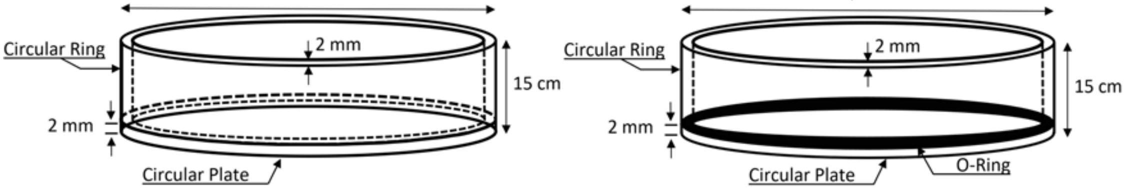

Figure 1. Schematic diagram of the dimensions of the plates with (right) and without (left) O-Ring between the circular base and the circular ring.

Figure 2 shows the plates under measurement used in this research.

Figure 2. Plates under measurement. Steel plate with O-ring (left), steel plate without O-ring (centre) and aluminium plate with O-ring (right).

3.2 Experimental SetUp

An experimental methodology based on the use of MLS signals was applied to obtain the parameters that let determine the radiation efficiency. This technique requires an input (the exciter that generates vibrations in the structure) and the outputs (accelerometers for measuring the levels of structural acceleration and a hydrophone for measuring the sound pressure level in the water). Figure 3 shows the diagram of the experimental set-up.

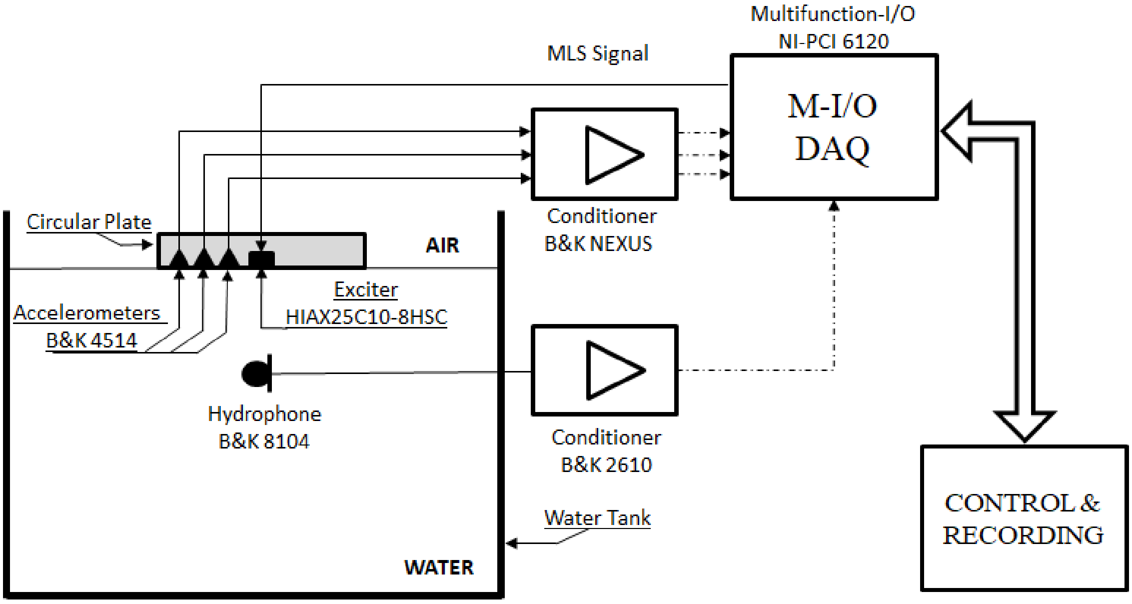

Figure 3. Diagram of the experimental Set-Up.

The excitation of the structure was carried out by means of a HIAX25C10-8HS electrodynamic type actuator, which generated the MLS signal provided by a NI-PCI 6120 National Instruments multifunction card. The excitation was carried out in the geometric centre of the circular base of the plates.

For the sound pressure acquisition, a Bruel & Kjael 8104 hydrophone was used. The acoustic signals measured were amplified by a Bruel & Kjael 2610 signal conditioner. The surface acceleration was measured using three piezoelectric accelerometers Bruel & Kjael model type 4514-B-002 with negligible weight. In this case, the signals were conditioned with a Bruel & Kjael Nexus signal conditioner. All the data was acquired by means of National Instruments multifunction card model NI-PCI 6120.

The software application used to control, manage and record the signals was developed using LabView©. The analysis of the measurement data was performed in MATLAB©.

3.2 Measurement procedure

The measurements were carried out at the facilities of the Consejo Superior de Investigaciones Científicas (CSIC) at Leonardo Torres Quevedo Institute in Madrid. The CSIC facilities have a water tank of 7.5 meters long, 4,5 meters width and 4.5 meters deep provided with two automatic-controlled positioners. Usually, one positioner is configured as an emitter (positioner 1) and the other as a receiver (positioner 2). However, during these experiments the positioner 1 was not used.

The Bruel & Kjael 8104 hydrophone was installed in positioner 2. In order to avoid acoustic diffraction problems, the hydrophone was sufficiently separated from the positioner using a metal rod longer than 2 meters. The positioner 2 was automatically moved to the measurement position by means of a software owned by the CSIC. The accelerometers were installed inside the vibrating structure in different positions and the exciter in the centre of the plate base.

The vibrating structures were positioned in the centre of the water tank, fixed by means of thin wires as in [12] to avoid the movement of the structures during the measurements and to simulate free-edge boundary conditions. During the installation of the structures over the water, it was considered that the tension of the strings should be enough to fix the structure guaranteeing that the circular base was water loaded.

Figure 4 shows an installed vibrating structure and the hydrophone positioned below it to measure the radiated acoustic pressure.

Figure 4. Fixing of the structure in the centre of the water tank (left) and detailed view of the hydrophone positioning (right).

The measurements of radiated sound pressure in water were made in centred vertical and horizontal planes through a previously planned grid covering a total space of 100 cm x 100 cm. This design allowed to cover the space under the plate and surrounding water. Each grid was composed of 121 measurement points with a distance between them of 10 mm on each direction. The horizontal plane was measured at 55 mm depth.

During the measurements, special care with deployed cabling was taken in order to avoid self-noise and unintentional movement of the plates.

4. RESULTS AND DISCUSSION

4.1. Surface velocity and radiated acoustic pressure

The average surface velocity was computed dividing the measured average acceleration by iw being i the imaginary unit and w the angular frequency.

Figure 6 shows the computed average surface velocity of each structure and the average radiated sound pressure in the bandwidth from 100 Hz to 10 kHz computed for the three structures under study.

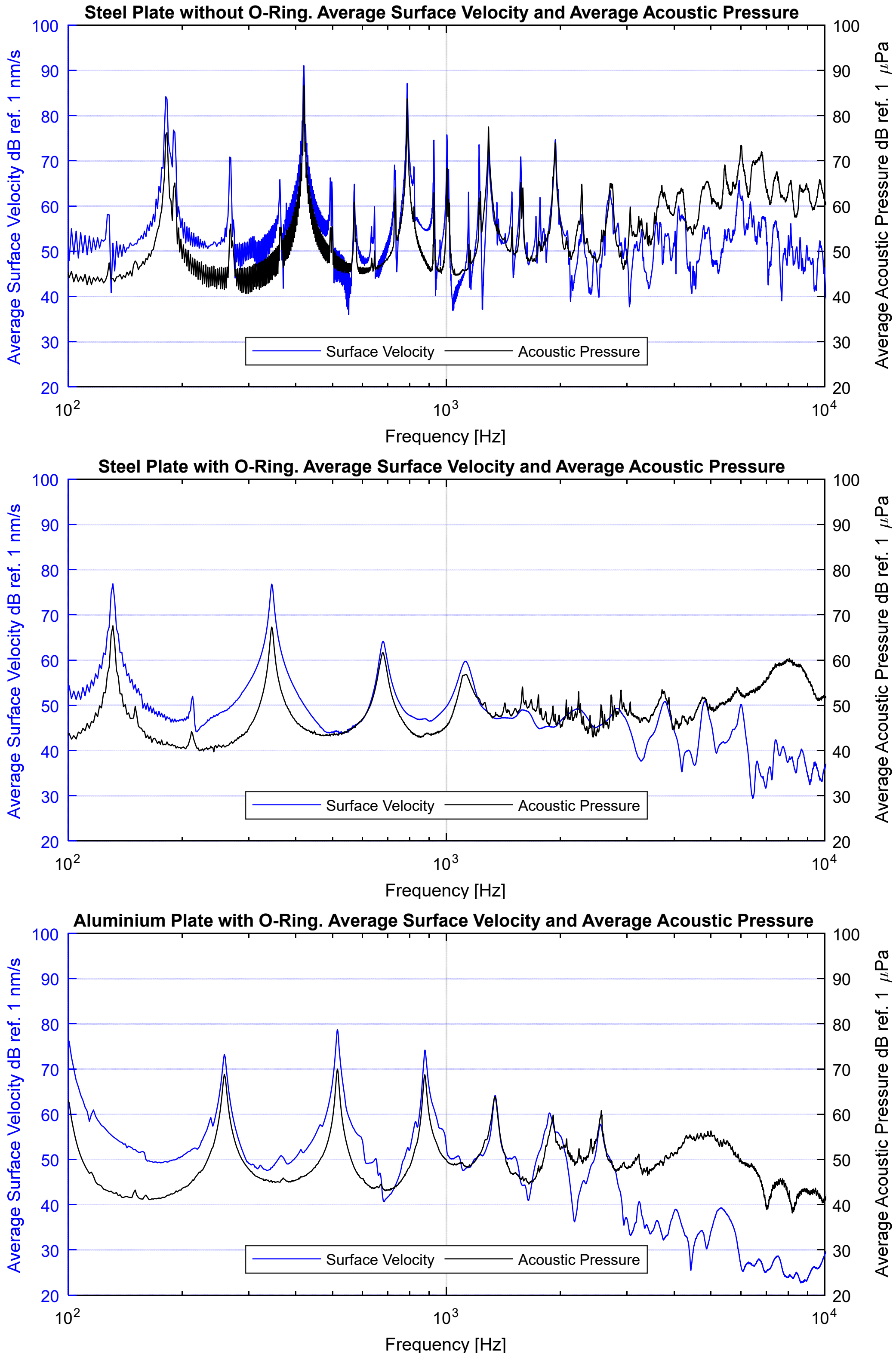

Figure 6. Average surface velocity and average radiated acoustic pressure of the steel structure without O-ring (up), steel structure with O-ring (centre) and aluminium structure with O-ring (down).

The frequencies of the modes were different for the three structures under study, not only due to the different type of material used to manufacture them, but also due to the method implemented to fix the circular base and the circular ring. The different types of boundary conditions, rigid or flexible, modified the vibration behaviour of the plates, decreasing the average surface velocity at high frequencies, from 3 kHz to 10 kHz, for structures with flexible boundary conditions.

The maximum peaks of the average acoustic pressure appeared for the steel structure without O-ring. In the high frequency band, the radiated acoustic pressure had a different trend depending on the material of the structures, increasing for the steel structures and decreasing for the aluminium structure.

4.2. Radiation efficiency analysis

The radiation efficiency was computed for each structure according to Equation 1 with water density, ρ, value of 1000 kg/m^3 and sound velocity, c, value of 1500 m/s. The total acoustic power was calculated using the far field approach by means of the Rayleigh integral according to the Equation 4. Figure 7 shows the computed radiation efficiency of the three structures under study in third octave bands.

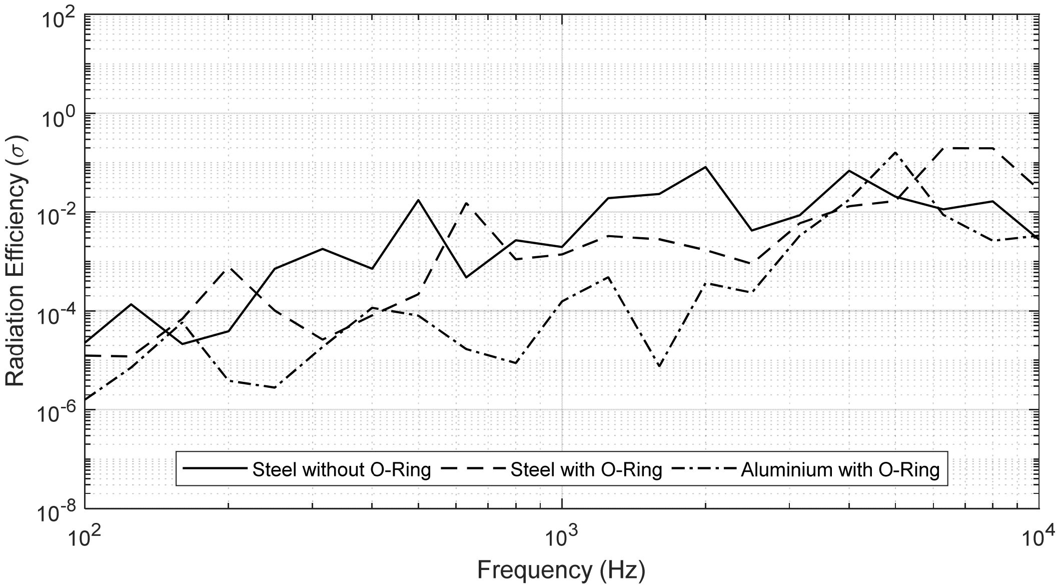

Figure 7. Radiation efficiency computed for each vibrating structure in third octave bands.

The analysis of the third octave bands of the radiation efficiency was divided into two main bandwidths: low-mid bandwidth covering the third octave bands from 100 Hz to 4 kHz and high bandwidth covering the third octave bands from 4 kHz to 10 kHz. In the low-mid bandwidth, the most efficient material was the steel. The rigid boundary conditions improved the radiation efficiency excepting in the band at 200 Hz and 625Hz.

In the high bandwidth, the flexible boundary conditions improved the efficiency of the steel. The most efficient structure was the plate manufactured of steel with O-Ring excepting in the third octave band at 5000 Hz. For this band, the plate manufactured with aluminium presented higher radiation efficiency.

5. CONCLUSIONS

In this paper the impulsive response measurement technique was used to investigate the average radiation efficiency of submerged circular plates made from different materials and with either rigid or flexible boundary conditions.

The used technique to measure the impulsive response was the MLS technique. This technique demonstrated high immunity against the external noise. This improvement provides a great advantage for measuring the impulse response in noisy environments. The direct method requires the generation of large pulses amplitudes to obtain high signal to noise ratio. This can be generated only through resonating transducers, which have limited bandwidth. For wide band applications high power speakers must be used and therefore the dimensions and cost of the system for measuring increases. The MLS technique can be implemented with small wide band transducers being an adequate technique for measuring the impulsive response for underwater applications.

By other hand, the FHT method provides a fast processing for the deconvolution of the impulsive response.

The analysis of the radiation efficiency computed by means of the measured impulsive response demonstrated that the most efficient material is the steel and the usage of flexible boundary conditions to fix the two parts of the structure improved the radiation efficiency at high frequencies.

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REFERENCES

1. J. W. S. Lord Rayleigh. “*Theory of Sound*”, New York: Dover Publications, second edition. 1945.

2. H. Lamb. “On the vibrations of an elastic plate in contact with water”. Proceedings of the Royal Society, London, A98, 205-216. 1921.

3. N. W. McLachlan. *“The accession to inertia of flexible discs vibrating in a fluid”*. Proceedings of the Physical Society, London, 44, 546-555. 1932.

4. Peake, W. H., Thurston, E. G. *“The lowest resonant frequency of a water-loaded circular plate”*. Proceedings of the Physical Society, London., 35, 170-182. 1923.Journal of the Acoustical Society of America, Vol 26, Nº 2, pp. 166-168. 1954.

5. Powell, J. H., Thurston, E. G. K. Kwak. *“On the frequency of vibration of circular diaphragms”*. Journal of Sound and Vibration, 201, 293-303. 1997.

6. M. Amabili and M. K. Kwak. *“Free vibration of circular plates coupled with liquids: revising the Lamb problem”*. Journal of Fluids and Structures, 10, 743-761. 1996.

7. M. K. Kwak. *“Hydroelastic vibration of circular plates”*. Journal of Sound and Vibration, 201, 293-303. 1997.

8. M. K. Kwak, K.C. Kim. *“Axisymmetric vibration of circular plates in contact with fluid”*. Journal of Sound and Vibration, 146, 381–389. 1991.

9. M. K. Kwak. *“Vibration of circular plates in contact with water”*. Transactions of the American Society of Mechanical Engineers, Journal of Applied Mechanics, 58, 480–483. 1991.

10. M. Amabili, G. Dalpiaz and C. Santolini. *“Free-edge circular plates vibrating in water”*. Modal Analysis: The International Journal of Analytical and Experimental Modal Analysis 10, 187-202. 1995.

11. M. K. Kwak and M. Amabili. *“Hydroelastic vibration of free-edge annular plates”*. Transactions of the ASME, Journal of Vibration and Acoustics 121, 26}32. 1998.

12. F. M. Espinosa and A. G. Gallego-Juárez. *“On the resonance frequencies of water-loaded circular plates”*. Journal of Sound and Vibration, 94, 217-222. 1984.

13. M. K. Kwak, S.B. Han. *“Effect of fluid depth on the hydroelastic vibration of free-edge circular plate”*. Journal of Sound and Vibration, 230, 171–185. 2000.

14. Davies, H. G. *“Acoustic radiation from fluid loaded rectangular plates”*. Massachusetts Institute of Technology: Acoustics and Vibration Laboratory. 1969.

15. Davies, H. G. *“Edge-mode radiation from fluid-loaded semi-infinite membranes”*. Journal of Sound and Vibration, 40 (2), 179-189. 1975.

16. Davies, H. G. *“Natural motion of a fluid-loaded semi-infinite membrane”*. Acoustical Society of America, 55, 213-219. 1974.

17. Berry, A. *“A new formulation for the vibrations and sound radiation of fluid-loaded plates with elastic boundary conditions”*. Quebec: University of Sherbrooke. 1994.

18. M. L. Rumerman. *“The effect of fluid loading on radiation efficiency”*. Journal of the Acoustical Society of America, 111, 75-79. 2002.

19. M. L. Rumerman. *“Radiation efficiencies for steel plates in water”*. Shock and Vibration, 14, 81-88. 2007.

20. Cheng, Z., Fan, J., Wan, B. *“Radiation efficiency of submerged rectangular plates”*. Applied Acoustics, 72, 150-157. 2012.

21. J. Borish and J. B. Angell, “*An efficient algorithm for measuring the impulse response using pseudorandom noise*,” J. Aud. Eng. Soc., vol. 31, no. 7/8, pp. 478–488, Aug. 1983.

22. Ning Xiang. “*Reciprocal maximum length sequence pairs for acoustical dual source measurements*”. Acoustical Society of America. November. 2002.